MAT 241 — Calculus III
Homework and Skills Overview

Avilez, Spring 2020
Calculus: Early Transcendentals, 7th Edition, James Stewart

How to use this outline

The homework is divided into chapters and sections according to the book. Each section is further divided into groups of exercises related by their focus on a particular skill or method.

Beside each group of exercises is a bar of three blocks that indicate where attention should be focused. In order from most to least important, these indications are

□□□ never attempted;

■□□ not adequately prepared;

■■□ need practice;

■■⧄ generally comfortable, but specific questions need practice; and

■■■ comfortable.

Homework pages are numbered according to the chapter (Ch), section (§), subsection (§§), and page of work (p) in that subsection, written as

$$Ch.\S.\S\S - p\,.$$

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Chapter 12

Vectors and the Geometry of Space

12.1 Three-Dimensional Coordinate Systems

Exercises to practice

1–38

☐☐☐ **12.1.1** **(1–4) Understand coordinates in three dimensions in terms of sets of points**

☐☐☐ **12.1.2** **(5–6) Understand the meaning of multivariable equations in $\mathbb{R}^2$ and $\mathbb{R}^3$**

☐☐☐ **12.1.3** **(7–8) Use the distance formula in three dimensions to classify triangles by their sides**

☐☐☐ **12.1.4** **(9) Determine the equation of a line in three dimensions and determine if it includes a third point**

☐☐☐ **12.1.5** **(10) Use the projections of a point onto the coordinate planes to find the distance from the point to those planes**

☐☐☐ **12.1.6** **(11–12) Find the equation for a sphere specified by its center and radius and determine its intersection with a coordinate plane**

☐☐☐ **12.1.7** **(13–14) Find the equation for a sphere with the given center that includes the given point**

Questions to ask

☐☐☐ **12.1.8** **(15–18) Characterize the sphere described by the given equation**

☐☐☐ **12.1.9** **(19) Prove the midpoint formula for a line segment in three dimensions**

☐☐☐ **12.1.10** **(20) Find the equation of a sphere that includes the two given points**

☐☐☐ **12.1.11** **(21) Find the equation of a sphere with the given center that intersects one of the coordinate planes**

☐☐☐ **12.1.12** **(22) Find the equation of a sphere with the given center that does not intersect any coordinate planes**

☐☐☐ **12.1.13** **(23–34) Describe the region of $\mathbb{R}^3$ represented by the equation**

☐☐☐ **12.1.14** **(35–38) Find an equation that represents the described region of $\mathbb{R}^3$**

12.2 Vectors

Exercises to practice

1–28

☐☐☐ **12.2.1** **(1) Differentiate vector quantities from scalar quantities**

☐☐☐ **12.2.2** **(2) Compare and contrast vectors and points**

☐☐☐ **12.2.3** **(3) Identify equal vectors in different positions**

☐☐☐ **12.2.4** **(4–5) Add and subtract vectors graphically using the head-to-tail method**

☐☐☐ **12.2.5** **(6) Add and subtract vectors multiplied by scalars**

☐☐☐ **12.2.6** **(7) Represent vectors as the sum or difference of other vectors by graphic analysis**

☐☐☐ **12.2.7** **(8) Add vectors that are at an angle to each other**

☐☐☐ **12.2.8** **(9–14) Represent a line segment as a vector in standard position (same magnitude and direction, but directed from the origin)**

☐☐☐ **12.2.9** **(15–18) Add vectors algebraically and draw their sum**

Questions to ask

☐☐☐ **12.2.10** **(19–22) Algebraically add and subtract vectors multiplied by scalars and find their magnitudes**

☐☐☐ **12.2.11** **(23–25) Normalize the given vector (find a unit vector in the same direction)**

☐☐☐ **12.2.12** **(26) Find a vector in the same direction as the one given, but with a specific magnitude (normalize and multiply by the given scalar)**

☐☐☐ **12.2.13** **(27–28) Find the standard angle (from the x-axis) of the given vector**

12.3 The Dot Product

Exercises to practice

1–44

Questions to ask

☐☐☐ **12.3.1 (1) Validate uses of the dot product with properties of dot products**

☐☐☐ **12.3.2 (2–12) Calculate the dot product**

☐☐☐ **12.3.3 (13) Show that the dot product of perpendicular vectors is zero, and that of unit vectors with themselves is one**

☐☐☐ **12.3.4 (14) Explain how the dot product can be used to calculate the magnitude of a linear combination**

☐☐☐ **12.3.5 (15–20) Use the dot product to find the angle between two vectors**

☐☐☐ **12.3.6 (21–22) Use the dot product to find all angles of the triangle bounded by the given vectors**

☐☐☐ **12.3.7 (23–24) Use the dot product to project one vector onto another and determine if they are orthogonal, parallel, or neither**

☐☐☐ **12.3.8 (25) Use the dot product of orthogonal vectors to determine if the triangle bounded by the given vectors has a right angle**

☐☐☐ **12.3.9 (26) Construct an equation with the dot product of two vectors such that one coordinate can be solved so that the angle between them has a specific measure**

☐☐☐ **12.3.10 (27) Construct a new basis for $\mathbb{R}^3$ with two sums of standard basis vectors, then find a third basis vector orthogonal to those**

☐☐☐ **12.3.11 (28) In a manner similar to (26), find two unit vectors that make a specific angle with the given vector**

☐☐☐ **12.3.12** **(29–30) Find the smallest angle between two lines by representing them with unit vectors (hint: the *smallest angle* requirement constrains the directions of the vectors)**

☐☐☐ **12.3.13** **(31–32) Find the smallest angle between the tangent lines of two curves at their point of intersection**

☐☐☐ **12.3.14** **(33–37) Find the direction cosines and direction angles of the given vector**

☐☐☐ **12.3.15** **(38) Given two direction angles for some vector, find the third**

☐☐☐ **12.3.16** **(39–44) Find the scalar and vector projections of one vector onto another**

☐☐☐ Properties of Dot Products

If $\mathbf{a}$, $\mathbf{b}$, and $\mathbf{c}$ are vectors in $\mathbb{R}^3$ and c is a scalar, then

1. $\mathbf{a} \cdot \mathbf{a} = |\mathbf{a}|^2$
2. $\mathbf{a} \cdot \mathbf{b} = \mathbf{b} \cdot \mathbf{a}$
3. $\mathbf{a} \cdot (\mathbf{b} + \mathbf{c}) = \mathbf{a} \cdot \mathbf{b} + \mathbf{a} \cdot \mathbf{c}$
4. $(c\mathbf{a}) \cdot \mathbf{b} = c(\mathbf{a} \cdot \mathbf{b}) = \mathbf{a} \cdot (c\mathbf{b})$
5. $\mathbf{0} \cdot \mathbf{a} = 0$

12.4 The Cross Product

Exercises to practice

1–38, 44

☐☐☐ **12.4.1** **(1–8) Find the cross product of two vectors and verify orthogonality (hint: the dot product of orthogonal vectors is zero)**

☐☐☐ **12.4.2** **(9–12) Find the resultant vector using properties of cross products**

☐☐☐ **12.4.3** **(13) Verify each expression using the properties of cross products**

☐☐☐ **12.4.4** **(14–15) Find the cross product and gives its direction (into or out of the page)**

☐☐☐ **12.4.5** **(16) Find the cross product and use the right hand rule to find the signs of its components**

☐☐☐ **12.4.6** **(17) Find both cross products of two vectors by changing the order**

☐☐☐ **12.4.7** **(18) Show that the cross product is not associative**

☐☐☐ **12.4.8** **(19–20) Find two orthogonal vectors to the two given vectors using the cross product**

Questions to ask

☐☐☐ **12.4.9** **(21–26) Prove properties of cross products**

☐☐☐ **12.4.10** **(27–28) Use the cross product to find the area of a parallelogram**

☐☐☐ **12.4.11** **(29–32) Use the cross product to find the normal of the plane through the given points; find the area of a triangle on that plane**

☐☐☐ **12.4.12** **(33–36) Use the cross product to find the volume of a parallelepiped**

☐☐☐ **12.4.13** **(44) Explain why the cross product of the two vectors cannot have one of its components be sign inverted (hint: prove by contradiction of orthogonality)**

Properties of Cross Products

If $\mathbf{a}$, $\mathbf{b}$, and $\mathbf{c}$ are vectors and c is a scalar, then

1. $\mathbf{a} \times \mathbf{b} = -\mathbf{b} \times \mathbf{a}$
2. $(c\mathbf{a}) \times \mathbf{b} = c(\mathbf{a} \times \mathbf{b}) = \mathbf{a} \times (c\mathbf{b})$
3. $\mathbf{a} \times (\mathbf{b} + \mathbf{c}) = \mathbf{a} \times \mathbf{b} + \mathbf{a} \times \mathbf{c}$
4. $(\mathbf{a} + \mathbf{b}) \times \mathbf{c} = \mathbf{a} \times \mathbf{c} + \mathbf{b} \times \mathbf{c}$
5. $\mathbf{a} \cdot (\mathbf{b} \times \mathbf{c}) = (\mathbf{a} \times \mathbf{b}) \cdot \mathbf{c}$
6. $\mathbf{a} \times (\mathbf{b} \times \mathbf{c}) = (\mathbf{a} \cdot \mathbf{c})\mathbf{b} - (\mathbf{a} \cdot \mathbf{b})\mathbf{c}$

12.5 Equations of Lines and Planes

Exercises to practice

1–74

Questions to ask

☐☐☐ **12.5.1** **(1) Support or contradict statements about the geometric relations between lines and planes in 3D space**

☐☐☐ **12.5.2** **(2–5) Find a vector equation and parametric equations for the line**

☐☐☐ **12.5.3** **(6–12) Find parametric equations and symmetric equations for the line**

☐☐☐ **12.5.4** **(13–14) Confirm or reject statements about the geometric relations of two lines**

☐☐☐ **12.5.5** **(15–16) Find equations for a line defined by a point and a direction vector; find intersections of that line with the coordinate planes**

☐☐☐ **12.5.6** **(17–18) Find equations for the line segment from one point to another**

☐☐☐ **12.5.7** **(19–22) Determine the geometric relations between two lines; find their point of intersection if it exists**

☐☐☐ **12.5.8** **(23–40) Find an equation of a plane**

☐☐☐ **12.5.9** **(41–44) Find intercepts to sketch the plane described by the equation**

☐☐☐ **12.5.10** **(45–47) Find the point of intersection between a line and a plane given their equations**

☐☐☐ **12.5.11** **(48) Find the point of intersection between a line through two points and a plane**

☐☐☐ **12.5.12** **(49) Find a direction vector for the line of intersection of two planes**

☐☐☐ **12.5.13** **(50) Find the angle between two planes**

☐☐☐ **12.5.14** **(51–56) Determine the geometric relations between two planes; find the angle between them if it is nontrivial**

☐☐☐ **12.5.15** **(57–58) Find parametric equations for the line of intersection of two planes and find the angle between them**

☐☐☐ **12.5.16** **(59–60) Find symmetric equations for the line of intersection of the two planes**

☐☐☐ **12.5.17** **(61–63) Find an equation for a plane that contains the given points**

☐☐☐ **12.5.18** **(64) Find the point of intersection of two lines and the plane that contains them**

☐☐☐ **12.5.19** **(65–66) Find parametric equations for a line through the given point that has a particular geometric relation to a plane**

☐☐☐ **12.5.20** **(67) Identify parallel and identical planes from their equations**

☐☐☐ **12.5.21** **(68) Identify parallel and identical lines from their equations**

☐☐☐ **12.5.22** **(69–70) Use the orthogonal projection to find the distance from a point to a line**

☐☐☐ **12.5.23** **(71–72) Use the plane normal to find the distance from a point to a plane**

☐☐☐ **12.5.24** **(73–74) Find the distance between parallel planes**

12.6 Cylinders and Quadric Surfaces

Exercises to practice

1–36

☐☐☐ **12.6.1** **(1–2) Make the connection between curves in $\mathbb{R}^2$ and surfaces in $\mathbb{R}^3$ (hint: consider *free variables*)**

☐☐☐ **12.6.2** **(3–8) Sketch a surface given its equation in $\mathbb{R}^3$**

☐☐☐ **12.6.3** **(9–10) Observe how the coefficients on each variable of a surface influence its orientation**

☐☐☐ **12.6.4** **(11–20) Plot cross-sectional views of a surface to identify its shape**

☐☐☐ **12.6.5** **(21–28) Match each equation with the graph of its surface and explain the connection**

☐☐☐ **12.6.6** **(29–36) Rearrange the equation to classify and sketch its surface (hint: standard form $ax^2+by^2+cz^2+\cdots$)**

Questions to ask

Chapter 13

Vector Functions

13.1 Vector Functions and Space Curves

Exercises to practice

1—30, 40–44

- ☐☐☐ **13.1.1 (1–2) Find the domain of a vector function (intersect the component domains)**
- ☐☐☐ **13.1.2 (3–6) Find the limit of a vector function**
- ☐☐☐ **13.1.3 (7–14) Sketch the curve traced by a vector function (indicate direction as parameter increases)**
- ☐☐☐ **13.1.4 (15–16) Sketch the projections of a vector function onto the coordinate planes**
- ☐☐☐ **13.1.5 (17–20) Find a vector equation and parametric equations for the line segment between two points**
- ☐☐☐ **13.1.6 (21–26) Match parametric equations with their graphs**
- ☐☐☐ **13.1.7 (27) Show that the parametric equation traces a curve on the surface of a cone and sketch the curve**
- ☐☐☐ **13.1.8 (28) Show that the parametric equation describes the intersection of two surfaces and sketch the curve**
- ☐☐☐ **13.1.9 (29) Find points of intersection between a vector function and a paraboloid**
- ☐☐☐ **13.1.10 (30) Find points of intersection between a helical vector function and a sphere**
- ☐☐☐ **13.1.11 (40–44) Find a vector function that represents the intersection of two surfaces**

Questions to ask

13.2 Derivatives and Integrals of Vector Functions

Exercises to practice

1–28, 35–42

☐☐☐ **13.2.1 (1–2) Sketch difference quotient vectors of a curve and find the derivative and unit tangent of the vector function**

☐☐☐ **13.2.2 (3–8) Find and sketch the derivative of a vector function**

☐☐☐ **13.2.3 (9–16) Find the derivative of a vector function**

☐☐☐ **13.2.4 (17–20) Find the unit tangent vector of a function at a specific point**

☐☐☐ **13.2.5 (21–22) Find the first and second derivatives and the unit tangent vector of a function**

☐☐☐ **13.2.6 (23–26) Find parametric equations for the tangent line to the curve of a parametric function**

☐☐☐ **13.2.7 (27) Find a vector equation for the tangent line to the curve of intersection of two cylinders at a specific point**

Questions to ask

☐☐☐ **13.2.8 (28) Find the point on a curve where the tangent line is parallel to a specific plane**

☐☐☐ **13.2.9 (35–42) Evaluate the integral of a vector function**

13.3 Arc Length and Curvature

Exercises to practice

1–10, 13–31, 47–48

☐☐☐ **13.3.1** **(1–6) Find the length of a curve**

☐☐☐ **13.3.2** **(7–10) Approximate the length of a curve with a calculator (*no calculators on exam*)**

☐☐☐ **13.3.3** **(13–14) Reparameterize a curve on arc length**

☐☐☐ **13.3.4** **(15) Find the point a specific distance along a curve (*use reparameterization*)**

☐☐☐ **13.3.5** **(16) Reparameterize a curve and give an analysis of its geometry**

☐☐☐ **13.3.6** **(17–20) Find the unit tangent and normal vectors of a curve and use Formula [9] in *Stewart* (p. 856) to find its curvature**

☐☐☐ **13.3.7** **(21–23) Use Theorem [10] in *Stewart* (p. 856) to find the curvature of a function**

☐☐☐ **13.3.8** **(24–26) Find the curvature of a function at a specific point**

Questions to ask

☐☐☐ **13.3.9** **(27–29) Use Formula [11] in *Stewart* (p. 857) to find the curvature of a function**

☐☐☐ **13.3.10** **(30–31) Find the point of maximum curvature of a function and analyze the end-behavior of its curvature**

☐☐☐ **13.3.11** **(47–48) Find the unit tangent, normal, and binormal vectors (see definitions on the following page)**

Tangent, Normal, and Binormal Vectors

The unit tangent vector $\mathbf{T}(t)$ is the direction vector tangent to the curve $\mathbf{r}(t)$

$$\mathbf{T}(t) = \frac{\mathbf{r}'(t)}{|\mathbf{r}'(t)|}$$

The **principal unit normal vector** $\mathbf{N}(t)$ (the **unit normal**) is orthogonal to the unit tangent vector $\mathbf{T}(t)$ and points in the direction of the change in $\mathbf{T}(t)$

$$\mathbf{N}(t) = \frac{\mathbf{T}'(t)}{|\mathbf{T}'(t)|}$$

The **binormal vector** is the unit vector orthogonal to both $\mathbf{T}$ and $\mathbf{N}$

$$\mathbf{B}(t) = \mathbf{T}(t) \times \mathbf{N}(t)$$

Curvature

The curvature κ (*kappa*) of a vector function is given by

$$\kappa = \left|\frac{d\mathbf{T}}{ds}\right| = \frac{|\mathbf{T}'(t)|}{|\mathbf{r}'(t)|} = \frac{|\mathbf{r}'(t) \times \mathbf{r}''(t)|}{|\mathbf{r}'(t)|^3}$$

See respectively Definition 8 (p. 855), Equation 9 (p. 856), and Theorem 10 (p. 856) in *Stewart* for more about these equations.

Exam 1 Recap

☐☐☐ **12.5.8 Constructing a perpendicular plane**

☐☐☐ **12.6.5 Sketching a hyperboloid of two sheets**

☐☐☐ **13.3.6 Multiple derivatives of a vector**

Chapter 14

Partial Derivatives

14.1 Functions of Several Variables

Exercises to practice

2, 6–36, 38–54, 59–64

- ☐☐☐ **14.1.1** **(2, 6–8) Evaluate a multivariable function and interpret the meaning of its inputs and outputs. Understand what spaces the function maps and what the points in those spaces represent**
- ☐☐☐ **14.1.2** **(9–12) Evaluate a multivariable function and find its domain and range**
- ☐☐☐ **14.1.3** **(13–22) Find and sketch the domain of a multivariable function**
- ☐☐☐ **14.1.4** **(23–31) Sketch a graph of a multivariable function**
- ☐☐☐ **14.1.5** **(32) Match a multivariable function to its graph**
- ☐☐☐ **14.1.6** **(33) Interpret a contour map to deduce the shape of its graph**
- ☐☐☐ **14.1.7** **(34–35) Use a contour map to estimate values of its function**
- ☐☐☐ **14.1.8** **(36) Interpret the spacing between lines on a contour map and choose the appropriate function whose slope matches that spacing**

Questions to ask

- ☐☐☐ **14.1.9** **(38) Sketch a contour map of the function represented by a surface**
- ☐☐☐ **14.1.10** **(39–42) Sketch a graph of the function represented by a contour map**
- ☐☐☐ **14.1.11** **(43–50) Sketch a contour map of a given function**
- ☐☐☐ **14.1.12** **(51–52) Sketch and compare the contour map and graph of a given function**

☐☐☐ **14.1.13** **(53–54) Sketch a contour map representing the function in a given application**

☐☐☐ **14.1.14** **(59–64) Match a function with its graph and contour map and explain the connections**

14.2 Limits and Continuity

Exercises to practice

1, 2, 5–24

☐☐☐ **14.2.1 (1–2) Explain the continuity or discontinuity of a given function**

☐☐☐ **14.2.2 (5–22) Find the limit of a multivariable function or show that the limit does not exist**

☐☐☐ **14.2.3 (23–24) Graph a function on a computer and interpret the graph to explain why the given limit does not exist**

Questions to ask

14.3 Partial Derivatives

Exercises to practice

3–8, 10–12, 15–44, 47–74

☐☐☐ **14.3.1** **(3–4) Estimate the partial derivatives of a function given a table of its values**

☐☐☐ **14.3.2** **(5–8) Use the graph of a function to determine the signs of its partial derivatives at a point**

☐☐☐ **14.3.3** **(10) Use the contour map of a function to estimate its partial derivatives at a point**

☐☐☐ **14.3.4** **(11–12) Sketch the curve of a given function with its tangent lines at a point (interpret the two partial derivatives as slopes)**

☐☐☐ **14.3.5** **(15–40) Find the first partial derivatives of a function**

☐☐☐ **14.3.6** **(41–44) Find the partial derivative of a function at a point**

☐☐☐ **14.3.7** **(47–50) Use implicit differentiation to find the partial derivatives of a function**

Questions to ask

☐☐☐ **14.3.8** **(51–52) Find the partial derivatives of a sum of functions, a product of functions and a quotient of functions (*hint:* what are the properties of a linear transformation?)**

☐☐☐ **14.3.9** **(53–58) Find the second partial derivatives of a function**

☐☐☐ **14.3.10** **(59–62) Verify Clairaut's Theorem ($u_{xy} = u_{yx}$, *Stewart* p. 907)**

☐☐☐ **14.3.11** **(64–72) Find the indicated higher-order partial derivatives of a function**

☐☐☐ **14.3.12** **(73) Use a table to estimate the values of certain higher-order partial derivatives of a function**

☐☐☐ **14.3.13 (74) Use level curves to determine the signs of certain higher-order partial derivatives of a function**

14.4 Tangent Planes and Linear Approximation

Exercises to practice

1–6, 11–20, 25–36

☐☐☐ **14.4.1** **(1–6) Find an equation of the tangent plane to a surface at a given point**

☐☐☐ **14.4.2** **(11–16) Explain why a function is differentiable at the given point and find the linearization $L(x, y)$ at that point**

☐☐☐ **14.4.3** **(17–18) Verify the given linear approximation at the point $(0, 0)$**

☐☐☐ **14.4.4** **(19–20) Use a linear approximation to estimate the value of a function at the given point**

☐☐☐ **14.4.5** **(25–30) Find the differential of a function**

☐☐☐ **14.4.6** **(31–32) Compare the values of Δz and the differential dz for a function on the given interval**

☐☐☐ **14.4.7** **(33–35) Use differentials to estimate a solution of a geometric problem**

☐☐☐ **14.4.8** **(36) Use differentials to estimate the error in a weather model due to measurement**

Questions to ask

14.5 The Chain Rule

Exercises to practice

1–34

☐☐☐ **14.5.1** **(1–16) Use the Chain Rule to find the derivative or partial derivative**

☐☐☐ **14.5.2** **(17–20) Use a tree diagram to write out the Chain Rule for the given cases**

☐☐☐ **14.5.3** **(21–26) Use the Chain Rule to find the partial derivative**

☐☐☐ **14.5.4** **(27–30) Use the implicit function theorem (Equation [6] in *Stewart*, p. 929) to find the derivative**

☐☐☐ **14.5.5** **(31–34) Use the expanded implicit function theorem (Equation [7] in *Stewart*, p. 930) to find the derivative**

Questions to ask

Exam 2 Recap

☐☐☐ **14.1.11** 45 **Draw a contour map with enough levels to show the behavior of a function**

☐☐☐ **14.4.1** 5 **Write the equation of the tangent plane of a function at a specified point**

Chapter 14

Partial Derivatives (cont.)

14.6 Directional Derivatives and the Gradient Vector

Exercises to practice

1–26, 28–34, 36, 41–46, 49, 50, 54–56

☐☐☐ **14.6.1** **(1–3) Estimate the derivative in a particular direction given discrete values for a function (see 14.3.1.3)**

☐☐☐ **14.6.2** **(4–6) Find the directional derivative of a function in a direction indicated by a given angle (use the cosine definition of the dot product)**

☐☐☐ **14.6.3** **(7–17, 19, 20) Find the gradient of a function and use it to find the derivative in the direction of a given vector**

☐☐☐ **14.6.4** **(18) Estimate the directional derivative of a function graphically from a vector diagram**

☐☐☐ **14.6.5** **(21–26) Find the magnitude and direction of the maximum rate of change of a function**
(*hint:* $D_{\mathbf{u}} = |\nabla f|\,|\mathbf{u}|\cos\theta$ is maximized when $\theta = 0$)

☐☐☐ **14.6.6** **(27) Prove that $-\nabla f$ is the steepest rate of descent**

☐☐☐ **14.6.7** **(28) Construct and solve an equation to find the directions of a particular rate of change of f**

Questions to ask

☐☐☐ **14.6.8** **(29) Construct and solve an equation to find all points where the directional derivative points in a certain direction**

☐☐☐ **14.6.9** **(30) Use the gradient to find the rate of change of depth in a model of a lake. Explain how the gradient relates to properties of the system modeled**

☐☐☐ **14.6.10** **(31) Use the gradient to find the rate of change of temperature in a model; then use the gradient to describe the shape of the model**

☐☐☐ **14.6.11** **(32–34) Use the gradient to find the direction and magnitude of the greatest increase and decrease of a modeled parameter**

☐☐☐ **14.6.12** **(36) On a contour map, draw curves of steepest descent from one point to another**

☐☐☐ **14.6.13** **(41–46) Find equations of (a) the tangent plane and (b) the normal line to a given surface at a specified point**

☐☐☐ **14.6.14** **(49–50) Find the gradient vector at a point and use it to find the tangent line to the level curve. Sketch the level curve, tangent line, and gradient vector**

☐☐☐ **14.6.15** **(54–56) Identify geometric relations of the tangent plane of the given surface**

14.7 Maximum and Minimum Values

Exercises to practice

1–18, 21–24, 29–36, 39–51

☐☐☐ **14.7.1** **(1–2) Determine the behavior of the partial derivatives of a function at a critical point**

☐☐☐ **14.7.2** **(3–4) Find the critical points of a function given its contour map**

☐☐☐ **14.7.3** **(5–18) Find the local maximum and minimum values and saddle point(s) (the extrema) of a function**

☐☐☐ **14.7.4** **(21–24) Use a graph and/or level curves to estimate the local extrema of a function, then use calculus to find the exact values**

☐☐☐ **14.7.5** **(29–36) Find the absolute maximum and minimum of a function on the given set**

☐☐☐ **14.7.6** **(39–42) Use calculus to minimize the distance between two points**

☐☐☐ **14.7.7** **(43–45) Use calculus to find solutions of the given arithmetic system**

☐☐☐ **14.7.8** **(46–51) Use calculus to find solutions of the given geometric system**

Questions to ask

14.8 Lagrange Multipliers

Exercises to practice

1, 3–21, 29–41

☐☐☐ **14.8.1** **(1) Refer to Lagrange's method to estimate the maximum and minimum values of a function f constrained by g. Explain your reasoning**

☐☐☐ **14.8.2** **(3–14) Use Lagrange multipliers to find the minimum and maximum values of a function subject to a given constraint**

☐☐☐ **14.8.3** **(15–18) Find extreme values of a function subject to multiple constraints**

☐☐☐ **14.8.4** **(19–21) Find extreme values of a function constrained by an inequality**

☐☐☐ **14.8.5** **(29–41) Use Lagrange multipliers to find alternate solutions to the exercises in 14.7.6–14.7.8 (page 35 of this outline)**

Questions to ask

Exam 3 Recap

Chapter 15

Multiple Integrals

15.1 Double Integrals over Rectangles

Exercises to practice

> No homework is assigned for this section, so this is just a general outline.

The *Riemann Sum* referenced in this section is defined as

$$\sum_{i=1}^{n} f(x_i^*)\Delta x$$

where x_i^* are the partitions we discussed in class, each of size Δx. If we let n go to infinity this gives the definite area integral

$$A = \int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i^*)\Delta x.$$

This is the ordinary case of summing rectangles with infinitely small width Δx and height $f(x)$ to find the area.

We can take the *Double Riemann Sum* to find the definite volume integral over a region R

$$V = \int\!\!\int_R f(x,y)\,dA = \lim_{m,n\to\infty} \sum_{i=1}^{m}\sum_{j=1}^{n} f(x_{ij}^*, y_{ij}^*)\Delta A.$$

The partitions in this case are three-dimensional, with area $\Delta A = \Delta x \Delta y$ and height given by f. As m and n grow to infinity we are dividing our region into more partitions with increasingly small ΔA; at the limit ΔA is the differential area dA. Each area is centered on the point $\left(x_{ij}^*, y_{ij}^*\right)$, where x^* is the midpoint between $x-1$ and x, and y^* between $y-1$ and y.

Questions to ask

☐☐☐ **15.1.1 (1–4) Use (a) a Riemann Sum and (b) the Midpoint Rule to estimate the volume enclosed by a function**

☐☐☐ **15.1.2 (5–6) Use the Midpoint Rule to estimate the double integral given a table of values**

☐☐☐ **15.1.3 (7) Without calculating, predict which is largest: the actual volume of a solid, the estimate found by a left-handed Riemann Sum, and the estimate found by a right-handed Riemann Sum**

☐☐☐ **15.1.4** **(8–9) Use the Midpoint Rule to estimate the double integral of a function given a contour map**

☐☐☐ **15.1.5** **(10) Use the Midpoint Rule to estimate the average value of a function given a contour map (hint: the average value of a continuous function is given by $\frac{1}{b-a}\int_a^b f(x)\,dx$)**

☐☐☐ **15.1.6** **(11–13) Evaluate a double integral by first identifying it as the volume of a solid**

☐☐☐ **15.1.7** **(14) Sketch the solid represented by a given double integral**

☐☐☐ **15.1.8** **(15–16) Use a calculator to estimate a double integral using the Midpoint Rule with a variety of parameters**

☐☐☐ **15.1.9** **(17) Apply the Fundamental Theorem of Calculus to a double integral to prove a given solution**

☐☐☐ **15.1.10** **(18) Substitute trigonometric functions in the solution to exercise 17 to prove a given relation**

15.2 Iterated Integrals

Exercises to practice

1–31, 35, 36

☐☐☐ **15.2.1** **(1–2) Find the definite integrals of a function, first with respect to x and then with respect to y (hint: variables aside from the variable of integration are integrated as constants)**

☐☐☐ **15.2.2** **(3–14) Calculate an iterated integral (hint: while *repeated integration* considers the integrals of one variable, *iterated integration* considers a different variable at each repetition, holding the other variables constant)**

☐☐☐ **15.2.3** **(15–22) Calculate a double integral (hint: the double integral considers an input region defined by two variables; write the variable of integration in terms of those variables, ordering them to give the simplest integration)**

☐☐☐ **15.2.4** **(23–24) Sketch the solid whose volume is given by an iterated integral**

☐☐☐ **15.2.5** **(25–31) Find the volume of a solid using an iterated integral on the given region. As in 15.2.3, choose the order of the variables of integration to give the simplest integration (according to *Fubini's Theorem,* we're free to choose the order for any continuous function)**

Questions to ask

☐☐☐ **15.2.6** **(35–36) Use a double integral to find the average value of a function of two variables over the given region (hint: the average value of a continuous function is given by $\frac{1}{b-a}\int_a^b f(x)\,dx$)**

15.3 Double Integrals over General Regions

Exercises to practice

1–10, 13–32, 35–38, 43–54, 59, 60

☐☐☐ **15.3.1 (1–6) Evaluate an iterated integral**

☐☐☐ **15.3.2 (7–10) Write a double integral as an iterated integral to evaluate it**

☐☐☐ **15.3.3 (13–14) Express a region D as a region of type I (bounded above and below by two continuous functions of x) and as a region of type II (bounded on either side by two continuous functions of y). Evaluate $\iint_D$ using both the type I and type II regions**

☐☐☐ **15.3.4 (15–16) Write a double integral as two iterated integrals using both orders of the variables of integration. Evaluate both integrals and choose the simpler one; explain your choice**

☐☐☐ **15.3.5 (17–22) Evaluate a double integral, using any of the above techniques to find the easiest route**

☐☐☐ **15.3.6 (23–32) Find the volume of a solid by describing it with a double integral on the given region**

Questions to ask

☐☐☐ **15.3.7 (35–36) Find the volume of a solid as the difference between two volumes (hint: volume is always a positive value)**

☐☐☐ **15.3.8 (37–38) Sketch the solid whose volume is described by a given iterated integral**

☐☐☐ **15.3.9 (43–48) Sketch the region of integration and use the sketch to choose the preferrable order of integration**

☐☐☐ **15.3.10 (49–54) The given integral is difficult to calculate. Reverse the order of integration and calculate**

□□□ **15.3.11 (59–60) Find the average value of f over the region D (see 15.2.6 on page 43)**

15.4 Double Integrals in Polar Coordinates

Exercises to practice

1–27, 29–32

☐☐☐ **15.4.1** **(1–4) Given a region R, choose rectangular or polar coordinates and set up the general form $\iint_R f(x,y)\,dA$ as an iterated integral in the chosen coordinates**

☐☐☐ **15.4.2** **(5–6) Sketch the region whose area is given by an integral in polar coordinates and evaluate the integral**

☐☐☐ **15.4.3** **(7–14) Evaluate a given integral by changing to polar coordiantes (hint: change the variables of integration *and* the limits of integration)**

☐☐☐ **15.4.4** **(15–18) Use a double integral to find the area of a polar region**

☐☐☐ **15.4.5** **(19–27) Use a double integral in polar coordiantes to find the volume of a solid**

☐☐☐ **15.4.6** **(29–32) Evaluate an iterated integral by converting to polar coordinates**

Questions to ask

15.5 Applications of Double Integrals

Exercises to practice

3–14

☐☐☐ **15.5.1** **(3–14) Interpret the region of integration as the area of a lamina and the function as its density. Integrate to find the mass and center of mass of the lamina**

Questions to ask

15.7 Triple Integrals

Exercises to practice

1–23, 27–36

☐☐☐ **15.7.1** **(1) Evaluate the integral in Example 1 on page 1018 in *Stewart*, using a different order of integration than in the example**

☐☐☐ **15.7.2** **(2) Evaluate a given integral three times using three different orders of dx, dy, and dz**

☐☐☐ **15.7.3** **(3–8) Evaluate a given triple integral in the order as written**

☐☐☐ **15.7.4** **(9–18) Evaluate a given triple integral by first choosing an order for the three variables**

☐☐☐ **15.7.5** **(19–22) Use a triple integral to find the volume of a solid (hint: integrate the density of the volume, and assume the density is 1 everywhere)**

☐☐☐ **15.7.6** **(23) Set up a triple integral for the volume of a described solid and use a table of integrals to evaluate it exactly**

Questions to ask

☐☐☐ **15.7.7** **(27–28) Sketch the solid whose volume is described by a given integral**

☐☐☐ **15.7.8** **(29–36) Express a triple integral in all six orders of integration: $\{dx\,dydz, dx\,dzdy\}$, $\{dy\,dxdz, dy\,dzdx\}$, and $\{dz\,dxdy, dz\,dydx\}$**

15.8 Triple Integrals in Cylindrical Coordinates

Exercises to practice

1–12, 15–24, 29, 30

☐☐☐ **15.8.1** **(1–2) Plot the point in cylindrical coordinates and find its rectangular coordinates**

☐☐☐ **15.8.2** **(3–4) Change from rectangular to cylindrical coordinates**

☐☐☐ **15.8.3** **(5–8) Describe in words the surface whose equation is given in cylindrical coordinates**

☐☐☐ **15.8.4** **(9–10) Change an equation from rectangular to cylindrical coordinates**

☐☐☐ **15.8.5** **(11–12) Sketch the solid described by inequalities in cylindrical coordinates**

☐☐☐ **15.8.6** **(15–16) Sketch the solid whose volume is described by a given integral in cylindrical coordinates. Evaluate the integral**

☐☐☐ **15.8.7** **(17–24, 29–30) Evaluate a triple integral by changing from rectangular to cylindrical coordinates**

Questions to ask

15.9 Triple Integrals in Spherical Coordinates

Exercises to practice

1–15, 17–30, 39–41

□□□ **15.9.1** **(1–2) Plot the point in spherical coordinates and find its rectangular coordinates**

□□□ **15.9.2** **(3–4) Change from rectangular to spherical coordinates**

□□□ **15.9.3** **(5–8) Describe in words the surface whose equation is given in spherical coordinates**

□□□ **15.9.4** **(9–10) Change an equation from rectangular to spherical coordinates**

□□□ **15.9.5** **(11–14) Sketch the solid described by inequalities in spherical coordinates**

□□□ **15.9.6** **(15) Write a system of inequalities in spherical coordinates to describe the solid given in rectangular coordinates**

□□□ **15.9.7** **(17–18) Sketch the solid whose volume is described by a given integral in spherical coordinates. Evaluate the integral**

Questions to ask

□□□ **15.9.8** **(19–20) Choose cylindrical or spherical coordinates and set up a triple integral over the volumetric region shown**

□□□ **15.9.9** **(21–30, 39–41) Evaluate a triple integral, changing from rectangular to spherical coordinates if necessary**

Exam 4 Recap

Chapter 16

Vector Calculus

16.1 Vector Fields

Exercises to practice

1–18, 21–24, 29–32

☐☐☐ **16.1.1** **(1–10) Sketch a vector field (see *Stewart*, figures 5 on p. 1057 and 9 on p. 1058)**

☐☐☐ **16.1.2** **(11–18) Match a vector field to the plot of its values and explain your reasoning**

☐☐☐ **16.1.3** **(21–24) Find the gradient vector field of a function**

☐☐☐ **16.1.4** **(29–32) Match a function with the plot of its vector field and explain your reasoning**

Questions to ask

16.2 Line Integrals

Exercises to practice

1–26, 29a, 30a, 39–41

☐☐☐ **16.2.1** **(1–16) Evaluate a line integral over a parametric function**

☐☐☐ **16.2.2** **(17–18) Determine the sign of a line integral over a vector field**

☐☐☐ **16.2.3** **(19–22) Evaluate a line integral over a vector field**

☐☐☐ **16.2.4** **(23–26) Use numerical approximation to evaluate a line integral over a vector field (*hint:* use MATLAB `int(f,a,b)`, Mathematica `Integrate[f, {x, a, b}]`, Symbolab `\int_a ^b f dx`, or TI-83 `Math▸9:fnInt`)**

☐☐☐ **16.2.5** **(29a, 30a) Evaluate a line integral over a vector field**

☐☐☐ **16.2.6** **(39–41) Evaluate a line integral in the context of work done by a force field**

Questions to ask

16.3 The Fundamental Theorem for Line Integrals

1–20, 23, 24, 28–34

Exercises to practice

☐☐☐ **16.3.1 (1–2) Estimate the line integral over a function given its contour map or a table of values**

☐☐☐ **16.3.2 (3–10) Determine whether a vector field is conservative; if it is, integrate its gradient to find a function with that gradient**

☐☐☐ **16.3.3 (11) Use the Fundamental Theorem to explain why three line integrals over distinct in a vector field have the same value**

☐☐☐ **16.3.4 (12–18) Interpret a vector field as the gradient of a function and find that function, then calculate a line integral over a curve in that field**

☐☐☐ **16.3.5 (19–20) Show that a given line integral is independent of path and evaluate it**

☐☐☐ **16.3.6 (23–24) Find the work done by a force field on an object moving between the given points**

☐☐☐ **16.3.7 (28) The gradient of a function is given as a vector field. Find curves that are not closed and which satisfy the given line integral**

Questions to ask

☐☐☐ **16.3.8 (29) Prove that a conservative vector field has continuous first-order partial derivatives**

☐☐☐ **16.3.9 (30) Use the proof from 16.3.8 (Exercise 29) to show that a given line integral is not independent of path**

☐☐☐ **16.3.10 (31–34) Determine whether a set is open, connected, and simply-connected**

16.4 Green's Theorem

Exercises to practice

1–14, 21

☐☐☐ **16.4.1** **(1–4) Evaluate a line integral directly and using Green's theorem**

☐☐☐ **16.4.2** **(5–10) Use Green's Theorem to evaluate a line integral along a positively-oriented curve**

☐☐☐ **16.4.3** **(11–14) Use Green's Theorem to evaluate a line integral along a curve that may not be positively-oriented**

☐☐☐ **16.4.4** **(21) Proof geometric relationships between points by expanding integrals formulated using Green's Theorem**

Questions to ask

16.5 Curl and Divergence

Exercises to practice

1–20, 23–29

☐☐☐ **16.5.1** **(1–8) Find the curl and divergence of a vector field**

☐☐☐ **16.5.2** **(9–11) Determine the sign of the divergence of an illustrated vector field and the direction if its curl, if not zero**

☐☐☐ **16.5.3** **(12) Determine whether expressions involving divergence and curl are meaningful (hint: consider the dot and cross products of a scalar)**

☐☐☐ **16.5.4** **(13–18) Determine whether a vector field is conservative; if it is, find the function whose gradient is that field**

☐☐☐ **16.5.5** **(19–20) Determine whether a vector field with the given curl exists; explain why or why not**

☐☐☐ **16.5.6** **(23–29) Prove an identity involving divergence and/or curl**

Questions to ask

16.6 Parametric Surfaces and their Areas

Exercises to practice

1–6, 13–26, 33–50

☐☐☐ **16.6.1** **(1–2) Determine whether the given points lie on a particular surface**

☐☐☐ **16.6.2** **(3–6) Identify a surface given its vector equation**

☐☐☐ **16.6.3** **(13–18) Match a vector equation with its graph and explain your reasoning. Determine which families of curves on that surface have one constant component**

☐☐☐ **16.6.4** **(33–38) Find an equation of the tangent plane to a given surface at a specific point**

☐☐☐ **16.6.5** **(39–50) Find the area of a surface by parameterizing its equation**

Questions to ask

16.7 Surface Integrals

Exercises to practice

5–32

☐☐☐ **16.7.1 (5–20) Evaluate a surface integral**

☐☐☐ **16.7.2 (21–32) Evaluate an integral over an oriented surface; that is, find the flux of a vector field across a surface**

Questions to ask

16.8 Stokes' Theorem

Exercises to practice

1–10, 11a, 12a, 13–15, 18, 19

☐☐☐ **16.8.1** **(1) Explain why two different given surfaces have equal integrals of curl**

☐☐☐ **16.8.2** **(2–6) Use Stokes' Theorem to evaluate the integral of the curl of a surface**

☐☐☐ **16.8.3** **(7–10) Use Stokes' Theorem to evaluate a line integral over a vector field**

☐☐☐ **16.8.4** **(11a, 12a) Use Stokes' Theorem to evaluate a line integral over a vector field along a curve of intersection**

☐☐☐ **16.8.5** **(13–15) Verifty that Stokes' Theorem is true for a given surface in a vector field**

☐☐☐ **16.8.6** **(18) Evaluate a line integral over a vector field along a curve on a specified surface**

☐☐☐ **16.8.7** **(19) Show that the curl integral of a field over a spherical surface is zero if the field satisfies Stokes' Theorem**

Questions to ask

16.9 The Divergence Theorem

Exercises to practice

1–14, 17, 18, 24

☐☐☐ **16.9.1** **(1–4) Verify the Divergence Theorem for a particular region in a vector field**

☐☐☐ **16.9.2** **(5–14) Use the Divergence Theorem to calculate a surface integral in a field; that is, calculate the flux of a field across a surface**

☐☐☐ **16.9.3** **(17) Use the Divergence Theorem to calculate the field integral on a surface defined by multiple equations**

☐☐☐ **16.9.4** **(18) Find the flux of a field aross a surface defined by multiple equations**

☐☐☐ **16.9.5** **(24) Use the Divergence Theorem to evaluate an integral on a spherical surface**

Questions to ask